**W8 InLab Activity: Name: Cole Bardin**

*first*

*last*

**Summer 2022**

**Problem 1: Review: Solve this DE using the Method of Undetermined Coefficients**

Consider the following linear, non-autonomous differential equation which describes a **cycloid**:

Above, the forcing term grows linearly in time.

**a.** Find the particular solution

A

**i.** Give the derivative of your guess:

0

**ii.** Give the double derivative of your guess:

**iii.** Solve for the unknown coefficients *A* and *B*.

Plugging into the full DE we find: (show work here.)

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so, the particular solution is:

**b.** Find the general solution to the **homogeneous DE**:

**i.** Give the characteristic equation.

**ii.** The roots are the complex conjugates:

+i

-i

and

**iii.** So, the general **homogeneous** solution is: (Use for the cosine term and for the sine.)

**c.** The complete solution to the nonhomogeneous DE is:

**d.** Find the coefficients and that match the initial conditions.

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and

**Problem 2: The Cycloid Revisited! Equations in Normal Form**

Consider the same differential equation for a **cycloid** seen in the previous problem.

**a.** Represent the system in **normal form** after defining and so that the **state vector** is:

and its derivative is

Give the matrix *A* and the vector so that our DE is equivalent to:

**b.** Give the initial value of the state vector:

**c.** Verify that is a particular solution to the DE:

**LHS:** **RHS:**

**d.** Verify that is a solution to the homogeneous DE:

**LHS:** **RHS:**

**e.** The general solution to the full DE is thus:

**f.** Find the specific solution matching the initial condition that: . That is, find and .

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and

**Note:** The top component gives the same solution as in the previous problem:

**Problem 3: The Cycloid Revisited - Solve the cycloid DE using the Laplace Transform**

Use Laplace Transforms to solve this differential equation for the cycloid.

**a.** Find the transform of the RHS forcing function .

**b.** Find the transform of the double derivative term on the LHS. Denote the transform of the unknown as .

**c.** Solve for the solution (in transform space) as a ratio of two polynomials in *s*.

**d.** Find the partial fraction expansion for

**i.** First find *B* using the Heaviside Cover-up Method.For free, you would find .

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**e.** From the partial fraction expansion for we see that and .

Thus, the full partial fraction expansion is:

Give the solution in the time domain.

Here's a plot of the solution in phase space with on the vertical axis and on the horizontal axis.

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